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**FLOW-INDUCED VIBRATION OF  
CIRCULAR CYLINDRICAL STRUCTURES**

**by**

**Shoei-Sheng Chen**

**BASE TECHNOLOGY**



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**ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS**

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Components Technology Division

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NOMENCLATURE ➔

NOMENCLATURE

a	Amplitude of harmonic oscillations
c	Velocity of sound
$C_m$	Added mass coefficient
$c_p$	Phase velocity
[C]	Damping matrix
$C_D$ ( $C_L$ )	Steady drag (lift) coefficient
$C_{Dj}$ ( $C_{Lj}$ )	Steady drag (lift) coefficient for jth cylinder
$C'_D$ ( $C'_L$ )	Periodic fluctuating drag (lift) coefficient
$C'_{Dj}$ ( $C'_{Lj}$ )	Periodic fluctuating drag (lift) coefficient for jth cylinder
$C_s, C_{sj}, C_{sp}$	Viscous damping coefficient of a structure
$C_v$	Viscous damping coefficient
D	Diameter of a cylinder (= 2R)
$D_h$	Hydraulic diameter
$D_o$	Diameter of outer cylinder (= $2R_o$ )
E	Modulus of elasticity
$E_j$	Modulus of elasticity for shell j
$E_p I_p, EI$	Flexural rigidity of cylinder
f	Oscillation frequency
$f_f$	Natural frequency in fluid
$f_s$	Frequency of vortex shedding
$f_v$	Natural frequency in vacuum
$f_{fq}$	Natural frequency of qth mode in fluid
$f_{vj}$	Natural frequency of jth cylinder in vacuum
F	Generalized force
g	Fluid force component
$g_j$	Fluid-force component in the x direction of jth cylinder
$g'_j$	Fluctuating fluid-force component in the x direction of jth cylinder

$g_{sp}$	Force per unit length
$G$	Generalized force or gap
$h$	Shell thickness
$h_j$	Fluid-force component in the $y$ direction of $j$ th cylinder or the wall thickness of the $j$ th shell
$h'_j$	Fluctuating fluid-force component in the $y$ direction of $j$ th cylinder
$i$	$\sqrt{-1}$
$I$	Moment of inertia
$k$	Wave number ( $= \omega/c$ )
$k_s$	Spring constant
$k_{sj}$	Spring constant for cylinder $j$
$k_f$	Fluid stiffness
$K$	Bulk modulus of fluid
$K_c$	Keulegan-Carpenter parameter
$[K]$	Stiffness matrix
$\lambda$	Length or axial wave length
$m$	Cylinder mass per unit length
$m'$	$m + m_a$
$m_j$	Cylinder mass per unit length of cylinder $j$
$m_p$	$= m_j$ for $j = 1$ to $N$ and $m_{p-N}$ for $p = N + 1$ to $2N$
$m_a$	Added mass
$[M]$	Mass matrix
$M_d$	Displaced mass of fluid or mass of fluid inside a tube
$M_c$	Mach number
$m_p$	Displaced mass of fluid per unit length of cylinder $j$
$M_k$	Kinetic Mach number
$N$	Number of cylinders in an array
$p$	Fluid pressure
$P$	Pitch

$\{Q\}$	generalized coordinates
$r, \theta, z$	Cylindrical coordinates
$\vec{r}$	Position vector
R	Radius of cylinder ( $= D/2$ ) or radius of curved pipes
$R_j$	Radius of cylinder j or shell j
Re	Reynolds number
$R_k$	Kinetic Reynolds number
$R_o$	Radius of outer cylinder
St	Strouhal number
t	Time
T	Period, axial tension, transverse pitch
TI	Turbulence intensity
u	Cylinder displacement or shell displacement in the axial direction
$\vec{u}$	Velocity vector
$u'$	Fluctuating velocity component
$u_j$	Cylinder displacement of jth cylinder in the x direction or axial displacement of jth shell
$u_p$	$= u_j$ for $p = 1$ to $N$ and $v_j$ for $p = N + 1$ to $2N$
U	Flow speed
$\bar{U}$	Mean flow velocity
$\vec{U}$	Flow velocity ( $= u_r \hat{e}_v, u_\theta \hat{e}_\theta, u_z \hat{e}_z$ )
$U_r$	Reduced flow velocity
v	$= (\frac{M_d}{EI})^{0.5} U_r$ or $(\frac{M_d}{EI})^{0.5} R U$ , or shell displacement in the tangential direction
$v_j$	Cylinder displacement of jth cylinder in the y direction or circumferential displacement of the jth shell
V	Volume
x,y,z	Cartesian coordinates
w	Shell displacement in the radial direction

$w_j$	Radial displacement of the jth shell
$\alpha_e$	Void fraction
$\alpha_{jk}, \beta_{jk}, \sigma_{jk}, \tau_{jk}$	Added mass coefficients
$\alpha'_{jk}, \beta'_{jk}, \sigma'_{jk}, \tau'_{jk}$	Fluid damping coefficients
$\alpha''_{jk}, \beta''_{jk}, \sigma''_{jk}, \tau''_{jk}$	Fluid stiffness coefficients
$\bar{\alpha}_{jk}, \bar{\beta}_{jk}, \bar{\sigma}_{jk}, \bar{\tau}_{jk}$	Added mass matrices
$\bar{\alpha}'_{jk}, \bar{\beta}'_{jk}, \bar{\sigma}'_{jk}, \bar{\tau}'_{jk}$	Fluid damping matrices
$\bar{\alpha}''_{jk}, \bar{\beta}''_{jk}, \bar{\sigma}''_{jk}, \bar{\tau}''_{jk}$	Fluid stiffness matrices
$\Upsilon_{pq}$	Added mass matrix
$\delta_s$	Scruton's number (mass-damping parameter)
$\zeta$	Damping ratio
$\zeta_n$	Modal damping ratio of the nth mode
$\zeta_f$	Damping ratio in fluid or fluid damping
$\zeta_v$	Damping ratio in vacuum
$\zeta_{fq}$	Damping ratio of qth mode in fluid
$\zeta_{vj}$	Damping ratio of jth cylinder
$\mu$	Viscosity
$\mu_p$	Eigenvalue of added mass matrix
$\mu_s$	Structural damping coefficient
$\nu$	Kinematic viscosity or Poisson's ratio
$\nu_c$	Dimensionless propagation constant
$\nu_j$	Poisson's ratio of the jth shell
$\rho$	Fluid density
$\rho_s$	Structure density
$\rho_j$	Density of shell j
$\kappa$	Complex wave number
$\tau$	Dimensionless axial tension
$\phi$	Velocity potential function
$\omega$	Circular frequency ( $= 2\pi f$ )

$\omega_f$	Natural frequency in radian in fluid ( $= 2\pi f_f$ )
$\omega_v$	Natural frequency in radian in vacuum ( $= 2\pi f_v$ )
$\omega_{vj}$	Natural frequency in radian of jth cylinder in vacuum
$\omega_{vpn}$	Natural frequency in radian of nth mode of pth cylinder in vacuum
$\omega_{fp}$	Natural frequency in radian of pth mode in fluid
$\omega_{fpn}$	Natural frequency in radian of coupled mode in fluid
$\bar{\omega}_{fj}$	Natural frequency in radian of uncoupled mode of j cylinder
$\Omega_D$ ( $\Omega_L$ )	Circular frequency associated with the drag (lift) forces
$\Omega_{Dj}$ ( $\Omega_{Lj}$ )	Circular frequency associated with parameter in the drag (lift) direction
$\Omega_n$	Dimensionless natural frequency of nth mode
$\phi$	Flow velocity potential
$\phi_{Dj}$ ( $\phi_{Lj}$ )	Phase angle associated with parameter in the drag (lift) direction
$\phi_n(z)$	Orthonormal function of nth mode
$\psi$	Flow velocity distribution function

### Subscripts

D (L)	Denote drag (lift) direction
f	Denote parameters related to fluid
j, k	Denote cylinder number j, k ( $j, k = 1 \text{ to } N$ ) <sup>21</sup>
m, n, l	0, 1, 2, ... $\infty$
N	Number of cylinders
p, q	1 to $2N$
s	Denote parameters related to structure
v	Denote parameters measured in vacuum

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**FLOW-INDUCED VIBRATION OF  
CIRCULAR CYLINDRICAL STRUCTURES**

by

**Shoei-sheng Chen**

**ABSTRACT**

Significant progress has been made in the understanding of vibration of circular cylinders subjected to flow, including development of analysis techniques and experiments on fluid forces, damping, stability boundary, and general structural response. This report summarizes the flow-induced vibration of circular cylinders in quiescent fluid, axial flow, and crossflow, and applications of the analytical methods and experimental data in design evaluation of various system components consisting of circular cylinders.

The information is organized into five general topic areas:

**Introduction:** Chapter 1 presents an overview of flow-induced vibration of circular cylinders. It includes examples of flow-induced vibration, various fluid force components, and nondimensional parameters as well as different excitation mechanisms. The general principles are applicable in different flow conditions.

**Quiescent Fluid:** Fluid inertia and fluid damping are discussed in Chapters 2, 3 and 4. Various flow theories are applied in different situations. The main results are the characterization of fluid effects on structural response. Emphasis is placed on isolated cylinders, multiple cylinders and circular cylindrical shells.

**Axial Flow:** Axial flow can cause subcritical vibration and instability. Chapter 5 summarizes the results for internal flow, while Chapter 6 considers the external flow. Both theoretical results and experimental data are examined.

**Crossflow:** Different excitation mechanisms can be dominant in different conditions for crossflow. Those include turbulent buffeting, acoustic resonance, vortex excitation, and dynamic instability. Appropriate excitation mechanisms are presented for a single cylinder, twin cylinders, and a group of cylinders.

**Design Considerations:** Applications of the general methods of analysis in the design evaluation of system components are described and various techniques to avoid detrimental vibration are presented. In addition, available design guides on this subject are discussed.

The results presented in this report are expected to be useful not only to designers but also researchers in this field.